Published by Institute of Physics Publishing for SISSA

Received: *December 10, 2005*
Revised: *April 12, 2006*
Accepted: *April 21, 2006*
Published: *May 8, 2006*

Quantum numbers of heavy neutrinos, tri-bi-maximal mixing through double seesaw with permutation symmetry, and comment on $\theta_{\rm sol} + \theta_c \simeq \frac{\pi}{4}$

Jihn E. Kim and Jong-Chul Park

School of Physics and Center for Theoretical Physics, Seoul National University
Seoul 151-747, Korea
E-mail: jekim@phyp.snu.ac.kr, jcpark@phya.snu.ac.kr

Abstract: Using the family symmetry, in the neutrino mass matrix we remove the Yukawa coupling (arising in the Dirac type mass between the heavy neutrinos and light lepton doublets) dependence in the double seesaw mechanism so that it is directly proportional to the mass matrix $m^{(nn)}$ of heavy Majorana neutrinos. The family symmetry is supposed to be broken spontaneously at high energy scale so that the neutrino mass matrix is given by the family symmetry at high energy scale. With the permutation symmetry S_3, we note a variety of possible mass hierarchies arising distinctly in neutrinos, charged leptons, $Q_{\rm em} = -\frac{1}{3}$ quarks, and $Q_{\rm em} = \frac{2}{3}$ quarks. Distinguishing these hierarchies, we obtain a relation between the CKM angles and the MNS angles. Finally, we comment on the approximate relation $\theta_{\rm sol} + \theta_c \simeq \frac{\pi}{4}$.

Keywords: GUT, Discrete and Finite Symmetries, Neutrino Physics.

Contents

1. Motivation **1**

2. Neutrino masses induced by heavy neutrinos **4**

3. Some properties of S_3 **6**
- 3.1 Representations 6
- 3.2 Tensor products 7

4. Mixing matrix of light and heavy neutrinos **10**
- 4.1 n-tuple maximal mixing 10
- 4.2 Light neutrino mass matrix from heavy neutrino mass matrix 11
- 4.3 Charged leptons 12
- 4.4 Tri-bi-maximal mixing 13

5. Relation $\theta_{\rm sol} + \theta_c \simeq \frac{\pi}{4}$ **13**
- 5.1 Up type quark masses 13
- 5.2 Relating MNS and CKM angles 14

6. Conclusion **18**

1. Motivation

Neutrino oscillations are parametrized by the MNS unitary matrix

$$U_{\rm MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \tag{1.1}$$

The $|\alpha\rangle \to |\beta\rangle$ transition amplitude in the time interval t is [1],

$$\langle \nu_\beta | \nu_\alpha \rangle_t = \sum_j U_{\alpha j} U^\dagger_{j\beta} e^{-iE_j t} \tag{1.2}$$

where $\alpha = \{e, \mu, \tau\}$ is the weak eigenstate index and $i = \{1, 2, 3\}$ is the mass eigenstate index. Then, in the vacuum for example the survival probability of flavor ν_α at high energy E is given by

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sum_{i,j} 4 |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2 \left(\frac{\Delta m^2_{ij}}{4E} t \right) \tag{1.3}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. However, neutrinos from the production point to the observing point pass through dense matter regions, noticeably changing the above survival probability in matter via the so-called MSW effect [2]. Currently, the disappearance data from atmospheric and solar neutrinos point toward the following mixing matrix,

$$U_{\rm MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}}e^{i\delta_3} & \times & 0 \\ \times & \times & \frac{1}{\sqrt{2}}e^{i\delta_2} \\ \times & \times & \frac{1}{\sqrt{2}}e^{i\delta_1} \end{pmatrix} \tag{1.4}$$

where $\times$ is unspecified. Motivated by this observation, recently a *tri-bi-maximal* mixing form has been suggested [3 – 5],

$$U_{\rm MNS} \simeq \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{1.5}$$

which is possible starting from some discrete symmetries such as the permutation symmetry S_3 [4] and tetrahedral symmetry A_4 [5]. Note that there does not exist a measurable CP phase in this form. Since the neutrino mixing matrix involves the unitary matrices diagonalizing charged lepton and neutrino mass matrices, one must consider both of these unitary matrices. With the S_3 symmetry, for example, the charged lepton and neutrino mass matrices are assumed to take different representations under S_3. One simple choice is assuming that charged leptons are singlets under the discrete group and the form (1.5) is obtained purely from the neutrino mass matrix. Another possibility is to assume a bi-maximal form for the neutrino mass and a tri-maximal form for the charged lepton mass as done in ref. [4]. Certainly, the latter choice is very appealing in the simplicity of explaining both the bi-maximal [6] and tri-maximal [4] structures in a single mixing matrix of (1.5).

But there exists another complication due to the hypothetical mechanism for generating neutrino masses. In the standard model(SM), there exist renormalizable couplings for charged lepton masses.[1] But to generate neutrino masses at the SM level, non-renormalizable dimension-5 couplings are needed. To obtain these through the seesaw mechanism, one needs heavier neutrinos, collectively represented as n. Thus, the above attractive proposal for the bi- and tri- structure has to be carefully addressed.

It is of utmost importance to relate the neutrino mass matrix and the n mass matrix, or there are too many parameters to be assumed to be specific values. In the seesaw scenario, there appear Yukawa couplings between singlet neutrinos and doublet neutrinos, which complicate a direct application of symmetry idea. In this regard, earlier Lindner et al. [8] studied the possibility of removing this Yukawa coupling dependence as 'screening of Dirac flavor structure'.

For this purpose, we introduce a family symmetry and use the double seesaw mechanism to relate neutrino and n mass matrices. In this process, we need two types of heavy neutrinos, collectively represented as n and N types and two continuous symmetries F_1

[1] Renormalizable and nonrenormalizable couplings are the effective ones at low energy.

and F_2. Specifically, the dependence of neutrino mass matrix on the Yukawa couplings involving N and lepton doublets are removed, which will be shown to be possible by a hierarchy of singlet vacuum expectation values(VEVs).

Another appealing phenomenological relation is the sum rule of the solar neutrino mixing, $\theta_{\rm sol}$, and the Cabibbo angle, θ_c,

$$\theta_{\rm sol}^{\rm exp} + \theta_c^{\rm exp} \simeq 33^{\rm o} + 13^{\rm o} \to \frac{\pi}{4}. \tag{1.6}$$

Already there exist many ideas trying to explain the above relation [6, 7] with GUTs and with the quark-lepton complementarity idea,[2] but most of them do not remove the Yukawa coupling dependence in the neutrino mass matrix. We find that ref. [8] independently observes the same kind of the removal of Yukawa coupling dependence under the phrase, 'screening of Dirac flavor structure'. In eq. (1.6), $\theta_{\rm sol}$ appears in the MNS matrix which is given by diagonalizing neutrino and charged lepton mass matrices,

$$U_{\rm MNS} = U_l^\dagger U_\nu, \tag{1.7}$$

and θ_c appears in the CKM matrix which is obtained by diagonalizing $Q=\frac23$ and $Q=-\frac13$ quark mass matrices,

$$U_{\rm CKM} = U_u^\dagger U_d. \tag{1.8}$$

To relate the mixing angles of the leptonic sector and the quark sector, one must unify leptons and quarks, or go beyond the SM to grand unified theories(GUTs) or the quark-lepton complementarity. Here, we will be interested in the above sum rule also, and employ the quark-lepton complementarity idea. But we will not discuss any specific model in detail.

At the SM level, there are four types of mass matrices: $Q_{\rm em}=-\frac13$ quark mass matrix $m^{(d)}$, $Q_{\rm em}=\frac23$ quark mass matrix $m^{(u)}$, $Q_{\rm em}=-1$ charged lepton mass matrix m_l, and $Q_{\rm em}=0$ neutrino mass matrix m_ν. GUTs relate some of these mass matrices. The well-known one is the SU(5) relation with a Higgs quintet generating both $Q_{\rm em}=-\frac13$ quark and $Q_{\rm em}=-1$ lepton mass matrices [10]. Then, we obtain a relation between four unitary matrices, $U_{\rm CKM}, U_{\rm MNS}$, and two unitary matrices U_ν and $U^{(u)}$ which diagonalize m_ν and $m^{(u)}$, respectively. Here, usually $U_{\rm CKM}$ and $U_{\rm MNS}$ are phenomenologically determined and U_ν and $U^{(u)}$ are given theoretically. Thus, in addressing the above questions, it is suggested that the relation arises naturally from a proposed discrete symmetry.

The four types of fermions have distinct mass hierarchies. The charged leptons and down type quarks have the similar pattern for masses, $m_{e,d} \ll m_{\mu,s} \sim \frac{1}{20} m_{\tau,b}$. The neutrino mass hierarchy is quite different from this,

$$\Delta m^2_{\nu\ ij} \ll \Delta m^2_{\nu\ jk} \tag{1.9}$$

where the LHS is for the solar neutrino oscillation and the RHS is for the atmospheric neutrino oscillation. Finally, the up type quark masses are distinct from any of the above patterns,

$$m_u \ll m_c \sim \frac{1}{150} m_t, \quad \text{or} \quad m_u, m_c \ll m_t. \tag{1.10}$$

[2]Some think that the quark-lepton complementary relation is just a numerical accident [9].

The pattern (1.9) hints two almost degenerate neutrinos compared to the other neutrino, and the pattern (1.10) hints two almost massless quarks compared to top quark. These observations can be used as an input in the mixing angle relation.

In the SM without a family symmetry, there results the CKM mixing since the Yukawa couplings for the up-type quark masses are given differently from those for the down-type quark masses. With a family symmetry, nonzero mixing angles can arise only after breaking the imposed family symmetry. Since low energy Yukawa couplings are given in terms of dimension 4 renormalizable couplings, we assume that the original high energy couplings are non-renormalizable. If we consider an S_3 family symmetry, the quark mixing can arise only after the S_3 symmetry is spontaneously broken differently for the up-type and down-type quark sectors. The same argument applies to the leptonic sectors also.

In section 2, we introduce two continuous quantum numbers F_1 and F_2 to relate the neutrino masses and the n masses via the double seesaw mechanism. In section 4, distinct patterns of neutrinos, charged leptons, $Q_{\rm em} = -\frac{1}{3}$ and $Q_{\rm em} = \frac{2}{3}$ masses are used to obtain the tri-bi-maximal MNS unitary matrix. In section 5, we try to connect $U_{\rm CKM}$ and $U_{\rm MNS}$ and obtain an approximate relation $\theta_{\rm sol} + \theta_c \simeq \frac{\pi}{4}$. Section 6 is a conclusion.

2. Neutrino masses induced by heavy neutrinos

The charged lepton and quark masses arise from the dimension four Yukawa couplings

$$-\mathcal{L}_Y = f^{(u)}_{IJ} u^{cI} H_2 q^J + f^{(d)}_{IJ} d^{cI} H_1 q^J + f^{(e)}_{IJ} e^{cI} H_1 l^J + \text{h.c.}\,, \tag{2.1}$$

where all the fermions are represented in terms of left-handed Weyl fields, q and l are the quark and lepton doublets, and we used the two Higgs doublet notation with hypercharges $Y(H_1) = -\frac{1}{2}$ and $Y(H_2) = \frac{1}{2}$. If we introduce only one Higgs doublet H_1, then we replace H_2 by $-i\sigma_2 H_1^*$. In eq. (2.1), the roman characters I, J represent the family indices. The quark and lepton masses between families are distinguished by the difference of their Yukawa coupling strengths. The smallness of Cabibbo angle in the two family case is due to the hierarchy $f_{12}, f_{21}, f_{11} \ll f_{22}$. For the three family case, we have $f_{22}, f_{i3}(i = 1, 2) \ll f_{33}$.

The gauge symmetry does not allow masses of neutrinos. To obtain neutrino mass, we must introduce more field(s). We adopt the seesaw idea of introducing SM singlet (neutral) heavy fermion(s), n. Then neutrino masses can arise through the seesaw mechanism, symbolically written as

$$m_\nu \sim \frac{(fv)^2}{\tilde{M}} \tag{2.2}$$

where v is the vacuum expectation value of the Higgs doublet H_2, and $\tilde{M}$ is the Majorana mass of the heavy neutrino(s). Because the Yukawa coupling appears as f^2 in the numerator, it is not expected that the single seesaw would remove the f^2 dependence. To remove the f^2 dependence, we must have the same f^2 appearing in the denominator also. For this purpose, a double seesaw is needed as depicted in figure 1. In another context, the double seesaw was considered in ref. [12], where however our attempt of removing the Yukawa coupling dependence was not tried. Also, a kind of $U(2)$ symmetry for dimension-5 neutrino mass operator was considered [13], which does not belong to our scheme either.

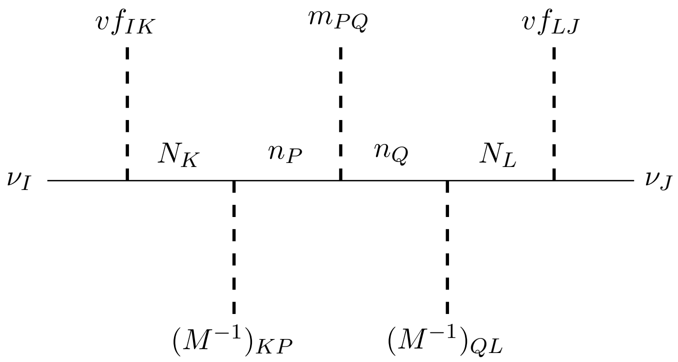

Figure 1: A double seesaw diagram with generic eigenvalues of fv, m and M have a hierarchy $M \gg m \gg fv$.

To relate Yukawa couplings appearing in the Dirac masses fv and M, we must introduce some symmetry. So let us introduce family quantum number Q_F. For each family, let us introduce the following chiral fermions

$$l_I \equiv \begin{pmatrix} \nu_I \\ l_I \end{pmatrix}, \quad l_I^c, \quad N_I, \quad n_I, \quad q_I, \quad u_I^c, \quad d_I^c \tag{2.3}$$

where N_I and n_I are neutral SU(2)×U(1)$_Y$ singlet heavy neutrinos and l and q are the lepton and quark doublets of the SM. One family is composed of 17 chiral fields, which together with Higgs multiplets can arise from the E_6 GUT with **27** [14] and trinification with $(\mathbf{3}, \mathbf{3}^*, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, \mathbf{3}^*) + (\mathbf{3}^*, \mathbf{1}, \mathbf{3})$ [15, 16].

The SM singlet neutral leptons can have bare masses unless they are forbidden by a symmetry. The lepton number is a good symmetry forbidding their bare masses. We assign the opposite lepton numbers to l_I and N_I. For n_I, we introduce another independent quantum number, say n-number. l_I and N_I do not carry the n number. The symmetry of leptons is $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{F_1} \times \mathrm{U}(1)_{F_2}$ where $\mathrm{U}(1)_{F_1} \times \mathrm{U}(1)_{F_2}$ is a continuous symmetry. To generate fermion masses, let us introduce the usual SU(2)×U(1)$_Y$ doublet Higgs field H_2 and SU(2)×U(1)$_Y$ singlet but U(1)$_{F_2}$ nonsinglet Ss. The lepton quantum numbers, F_1 and F_2, are

	l	e^c	N	n	S_1	S_2	H_2
F_1	1	−1	−1	1	−2	0	0
F_2	0	0	0	1	−2	−1	0

(2.4)

Consistently with the quantum numbers of (2.4), we can write the renormalizable Yukawa couplings involving singlet leptons as

$$-\mathcal{L} = f_{IJ}^{(lN)} N^I H_2 l^J + f_{IJ}^{(Nn)} N^I n^J S_2 + f_{IJ}^{(nn)} n^I n^J S_1 + \text{h.c.} \tag{2.5}$$

For a family symmetry, we require that f_{IJ} are the same if l, e^c, N and n belong to the same family, i.e.

$$f_{IJ}^{(lN)} = f_{IJ}^{(Nn)} \rightarrow \tag{2.6}$$

$$? = f_{IJ}^{(nn)}. \tag{2.7}$$

The above family symmetry relations may appear from the underlying theory at high energies. In this paper, however, we do not construct explicitly an underlying high energy model where the Yukawa couplings are generated by some important nonrenormalizable operators. The relation (2.6) for complex Dirac masses can be differentiated in principle from the relation (2.7) for real Majorana masses. But the family symmetry can be achieved by assigning l and n in the same multiplet. Note that the F_2 quantum numbers of l and n in (2.4) are different; thus we interpret F_2 as a $U(1)$ subgroup of a unifying group so that l and n can be put in the same representation of the unifying group.

The double seesaw diagram of figure 1 gives neutrino masses. We can see immediately that for f^2 to appear in the denominator, M of figure 1 can be taken to be much larger than those of m. However, it is known that even without this restriction the Yukawa coupling dependence disappears [8]. However, an intuitive understanding of this phenomenon is most transparent in the limit

$$V_2 \gg V_1 \gg v, \tag{2.8}$$

where $V_1 = \langle S_1 \rangle, V_2 = \langle S_2 \rangle$, and $v = \sqrt{2}\langle H_2^0 \rangle$. In this case, figure 1 gives the f^2 independent neutrino mass

$$\begin{aligned} m_{IJ}^{\nu} &= (vf_{IK})(M^{-1})_{KP}(m_{PQ})(M^{-1})_{QL}(vf_{LJ}) \\ &= \frac{v^2}{2}\frac{V_1}{V_2^2}\, f_{IK}^{(lN)}(f^{(Nn)})^{-1}_{KP} f_{PQ}^{(nn)}(f^{(Nn)})^{-1}_{QL} f_{LJ}^{(lN)} \\ &= \frac{v^2 V_1}{2V_2^2} f_{IJ}^{(nn)} \end{aligned} \tag{2.9}$$

where we used (2.6).

3. Some properties of S_3

If the family symmetry applied to the up-type and down-type quarks are identical, the CKM matrix would be diagonal. Therefore, it is necessary that the family symmetry is spontaneously broken differently for the up-type and down-type quark sectors. Let us briefly review how the S_3 symmetry can be broken differently for the up and down type quarks. The same strategy is applied to the leptons also. There is a long list of references on S_3, some of which are given in [18, 19].

3.1 Representations

S_3 is a permutation symmetry of three objects, which can be conveniently represented as permutations of three vertical points of equilateral triangle, $\mathbf{A} \sim (1, 0), \mathbf{B} \sim (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ and $\mathbf{C} \sim (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$. In the complexified coordinate, $(x + iy, x - iy)$, these points are

JHEP05(2006)017

represented as

$$\mathbf{A} \sim \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \mathbf{B} \sim \begin{pmatrix} \omega \\ \omega^2 \end{pmatrix}, \ \mathbf{C} \sim \begin{pmatrix} \omega^2 \\ \omega \end{pmatrix} \tag{3.1}$$

where ω is a cube root of unity, $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. The permutation operation of three objects is

$$\begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ i & j & k \end{array} \tag{3.2}$$

where $\{ijk\}$ is a permutation of $\{123\}$. The operation (3.2) is simply written as (ijk). Then, the six operations of S_3 are represented as

$$(123) \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ (231) \sim \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \ (312) \sim \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} \tag{3.3}$$

$$(132) \sim \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ (321) \sim \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \ (213) \sim \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \tag{3.4}$$

From three objects **A,B,C**, we can construct a singlet $\mathbf{S} \sim (\mathbf{A} + \mathbf{B} + \mathbf{C})$. The other remaining combinations form a doublet with components $\mathbf{D}^{\uparrow} \sim (\mathbf{A} + \omega^{\mathbf{2}}\mathbf{B} + \omega\mathbf{C})$ and $\mathbf{D}^{\downarrow} \sim (\mathbf{A} + \omega\mathbf{B} + \omega^{\mathbf{2}}\mathbf{C})$. Explicitly, we can show that

$$\mathbf{D}^{\uparrow} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{D}^{\downarrow} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

3.2 Tensor products

Consider a tensor product from two doublets of S_3,

$$\Psi_X = \begin{pmatrix} \psi_X^1 \\ \psi_X^2 \end{pmatrix}, \ \Psi_Y = \begin{pmatrix} \psi_Y^1 \\ \psi_Y^2 \end{pmatrix}. \tag{3.5}$$

The product representation $\Psi_X \otimes \Psi_Y$ has four elements which have the transformation following properties under S_3

$$\mathbf{1} \sim (\psi_X^1\psi_Y^2 + \psi_X^2\psi_Y^1), \ \mathbf{1'} \sim (\psi_X^1\psi_Y^2 - \psi_X^2\psi_Y^1), \tag{3.6}$$

and

$$\mathbf{2} \sim \begin{pmatrix} \psi_X^2\psi_Y^2 \\ \psi_X^1\psi_Y^1 \end{pmatrix}. \tag{3.7}$$

Repeating the multiplication rule (3.7), one can construct a singlet from three doublets of S_3 as

$$\psi_X^1\psi_Y^1\psi_Z^1 \pm \psi_X^2\psi_Y^2\psi_Z^2. \tag{3.8}$$

Including the above 2×2 tensor product, a dyadic is constructed from two triplets($\mathbf{3{=}S}$ $\mathbf{+D}$), $\mathbf{S_1}+\mathbf{D_1}$ and $\mathbf{S_2}+\mathbf{D_2}$ where

$$\mathbf{3_1}:\ \mathbf{S_1}=\frac{1}{3}(f_1+f_2+f_3),\ \mathbf{D_1}=\frac{1}{3}\begin{pmatrix} f_1+\omega^2 f_2+\omega f_3\\ f_1+\omega f_2+\omega^2 f_3\end{pmatrix} \tag{3.9}$$

$$\mathbf{3_2}:\ \mathbf{S_2}=\frac{1}{3}(f_1'+f_2'+f_3'),\ \mathbf{D_2}=\frac{1}{3}\begin{pmatrix} f_1'+\omega^2 f_2'+\omega f_3'\\ f_1'+\omega f_2'+\omega^2 f_3'\end{pmatrix}. \tag{3.10}$$

Let the two index S_3 representation be Φ_{ij} which transforms as a diadic

$$\Phi_{ij}\sim\phi_i\otimes\phi_j'. \tag{3.11}$$

Similarly with (3.9,3.10), we define singlets and doublets with ϕs,

$$\begin{aligned}&\mathbf{1}_\phi:\ \frac{1}{3}(\phi_1+\phi_2+\phi_3)\to\xi_1,\\ &\mathbf{2}_\phi:\ \frac{1}{3}\begin{pmatrix}\phi_1+\omega^2\phi_2+\omega\phi_3\\ \phi_1+\omega\phi_2+\omega^2\phi_3\end{pmatrix}\to\begin{pmatrix}\xi_2\\ \xi_3\end{pmatrix}\end{aligned} \tag{3.12}$$

$$\begin{aligned}&\mathbf{1}_{\phi'}:\ \frac{1}{3}(\phi_1'+\phi_2'+\phi_3')\to\xi_1',\\ &\mathbf{2}_{\phi'}:\ \frac{1}{3}\begin{pmatrix}\phi_1'+\omega^2\phi_2'+\omega\phi_3'\\ \phi_1'+\omega\phi_2'+\omega^2\phi_3'\end{pmatrix}\to\begin{pmatrix}\xi_2'\\ \xi_3'\end{pmatrix}\end{aligned} \tag{3.13}$$

We can introduce S_3 representations having two indices following the transformation rules of dyadic (3.12–3.13) for two-index singlets and two-index doublets. The nine components of the dyadic made of (3.12,3.13) split into the following S_3 multiplets, which constitute the representations of Φ,

$$S_3=\xi_1\xi_1',\quad S_4=\xi_2\xi_3'+\xi_3\xi_2',\quad S'=\xi_2\xi_3'-\xi_3\xi_2', \tag{3.14}$$

$$D_3=\xi_1\begin{pmatrix}\xi_2'\\ \xi_3'\end{pmatrix},\ D_4=\begin{pmatrix}\xi_2\\ \xi_3\end{pmatrix}\xi_1',\ D_5=\begin{pmatrix}\xi_3\xi_3'\\ \xi_2\xi_2'\end{pmatrix}. \tag{3.15}$$

Below, we will use predominantly the dyadic symbols written with ξs.

The renormalizable Yukawa coupling for quark masses are assumed to arise from non-renormalizable dimension 5 operators at the Planck scale

$$\sim\frac{1}{M_{Pl}}f_i f_j' H\langle\Phi_{ij}\rangle \tag{3.16}$$

where f_i is the symbol for a fermion, i,j are the labels of the permutation symmetry, H is a Higgs doublet which does not carry a family index, and Φ_{ij} is the two-index scalar field. Since H is a singlet of the family group, we can consider the relevant couplings presented in Table 1 with coupling constants λs, which are interpreted as coupling times $\langle H^0\rangle/M_{Pl}$. There are fifteen couplings.

Depending on the direction of VEVs, the permutation symmetry is broken.

JHEP05(2006)017

	$\mathbf{1}$	$\mathbf{1'}$
$\mathbf{S_1S_2S_3}$	$\lambda_1(f_1+f_2+f_3)(f'_1+f'_2+f'_3)\xi_1\xi'_1$	
$\mathbf{S_1S_2S_4}$	$\lambda_2(f_1+f_2+f_3)(f'_1+f'_2+f'_3)(\xi_2\xi'_3+\xi_3\xi'_2)$	
$\mathbf{S_1S_2S'}$		$\lambda_3(f_1+f_2+f_3)(f'_1+f'_2+f'_3)(\xi_2\xi'_3-\xi_3\xi'_2)$
$\mathbf{D_1D_2S_3}$	$\lambda_4[(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega f'_2+\omega^2 f'_3)$ $+(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega^2 f'_2+\omega f'_3)]\xi_1\xi'_1$	$\lambda_4[(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega f'_2+\omega^2 f'_3)$ $-(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega^2 f'_2+\omega f'_3)]\xi_1\xi'_1$
$\mathbf{D_1D_2S_4}$	$\lambda_5[(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega f'_2+\omega^2 f'_3)$ $+(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega^2 f'_2+\omega f'_3)](\xi_2\xi'_3+\xi_3\xi'_2)$	$\lambda_5[(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega f'_2+\omega^2 f'_3)$ $-(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega^2 f'_2+\omega f'_3)](\xi_2\xi'_3+\xi_3\xi'_2)$
$\mathbf{D_1D_2S'}$	$\lambda_6[(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega f'_2+\omega^2 f'_3)$ $-(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega^2 f'_2+\omega f'_3)](\xi_2\xi'_3-\xi_3\xi'_2)$	$\lambda_6[(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega f'_2+\omega^2 f'_3)$ $+(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega^2 f'_2+\omega f'_3)](\xi_2\xi'_3-\xi_3\xi'_2)$
$\mathbf{S_1D_2D_3}$	$\lambda_7[(f_1+f_2+f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_1\xi'_3$ $+(f_1+f_2+f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_1\xi'_2]$	$\lambda_7[(f_1+f_2+f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_1\xi'_3$ $-(f_1+f_2+f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_1\xi'_2]$
$\mathbf{S_1D_2D_4}$	$\lambda_8[(f_1+f_2+f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_3\xi'_1$ $+(f_1+f_2+f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_2\xi'_1]$	$\lambda_8[(f_1+f_2+f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_3\xi'_1$ $-(f_1+f_2+f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_2\xi'_1]$
$\mathbf{S_1D_2D_5}$	$\lambda_9[(f_1+f_2+f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_2\xi'_2$ $+(f_1+f_2+f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_3\xi'_3]$	$\lambda_9[(f_1+f_2+f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_2\xi'_2$ $-(f_1+f_2+f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_3\xi'_3]$
$\mathbf{D_1S_2D_3}$	$\lambda_{10}[(f_1+\omega^2 f_2+\omega f_3)(f'_1+f'_2+f'_3)\xi_1\xi'_3$ $+(f_1+\omega f_2+\omega^2 f_3)(f'_1+f'_2+f'_3)\xi_1\xi'_2]$	$\lambda_{10}[(f_1+\omega^2 f_2+\omega f_3)(f'_1+f'_2+f'_3)\xi_1\xi'_3$ $-(f_1+\omega f_2+\omega^2 f_3)(f'_1+f'_2+f'_3)\xi_1\xi'_2]$
$\mathbf{D_1S_2D_4}$	$\lambda_{11}[(f_1+\omega^2 f_2+\omega f_3)(f'_1+f'_2+f'_3)\xi_3\xi'_1$ $+(f_1+\omega f_2+\omega^2 f_3)(f'_1+f'_2+f'_3)\xi_2\xi'_1]$	$\lambda_{11}[(f_1+\omega^2 f_2+\omega f_3)(f'_1+f'_2+f'_3)\xi_3\xi'_1$ $-(f_1+\omega f_2+\omega^2 f_3)(f'_1+f'_2+f'_3)\xi_2\xi'_1]$
$\mathbf{D_1S_2D_5}$	$\lambda_{12}[(f_1+\omega^2 f_2+\omega f_3)(f'_1+f'_2+f'_3)\xi_2\xi'_2$ $+(f_1+\omega f_2+\omega^2 f_3)(f'_1+f'_2+f'_3)\xi_3\xi'_3]$	$\lambda_{12}[(f_1+\omega^2 f_2+\omega f_3)(f'_1+f'_2+f'_3)\xi_2\xi'_2$ $-(f_1+\omega f_2+\omega^2 f_3)(f'_1+f'_2+f'_3)\xi_3\xi'_3]$
$\mathbf{D_1D_2D_3}$	$\lambda_{13}[(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_1\xi'_3$ $+(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_1\xi'_2]$	$\lambda_{13}[(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_1\xi'_3$ $-(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_1\xi'_2]$
$\mathbf{D_1D_2D_4}$	$\lambda_{14}[(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_3\xi'_1$ $+(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_2\xi'_1]$	$\lambda_{14}[(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_3\xi'_1$ $-(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_2\xi'_1]$
$\mathbf{D_1D_2D_5}$	$\lambda_{15}[(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_2\xi'_2$ $+(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_3\xi'_3]$	$\lambda_{15}[(f_1+\omega f_2+\omega^2 f_3)(f'_1+\omega f'_2+\omega^2 f'_3)\xi_2\xi'_2$ $-(f_1+\omega^2 f_2+\omega f_3)(f'_1+\omega^2 f'_2+\omega f'_3)\xi_3\xi'_3]$

Table 1: Singlet combinations from $\mathbf{3_1} = \mathbf{S_1} + \mathbf{D_1}, \mathbf{3_2} = \mathbf{S_2} + \mathbf{D_2}$ and $\Phi_{ij} = \mathbf{S_3} + \mathbf{D_3} + \mathbf{D_4} + \mathbf{S_4} + \mathbf{S'} + \mathbf{D_5}$. The overall factor $\frac{1}{3}$ is omitted.

If the couplings are the same $\lambda_1 = \cdots = \lambda_{15} = \lambda$, the mass matrix takes the following form for the $\mathbf{1} + \mathbf{1'}$ coupling

$$\mathbf{1} + \mathbf{1'} : \frac{\lambda}{9} \begin{pmatrix} \begin{matrix} 3\xi_1\xi'_1 + 6\xi_2\xi'_3, \\ +6(\xi_1\xi'_3 + \xi_2\xi'_2 + \xi_3\xi'_1), \end{matrix} & (1+2\omega)(\xi_1\xi'_1 + 2\xi_2\xi'_3), & (1+2\omega^2)(\xi_1\xi'_1 + 2\xi_2\xi'_3) \\ (1+2\omega^2)(\xi_1\xi'_1 + 2\xi_2\xi'_3), & \begin{matrix} 3\xi_1\xi'_1 + 6\xi_2\xi'_3 \\ +6\omega^2(\xi_1\xi'_3 + \xi_2\xi'_2 + \xi_3\xi'_1), \end{matrix} & (1+2\omega)(\xi_1\xi'_1 + 2\xi_2\xi'_3) \\ (1+2\omega)(\xi_1\xi'_1 + 2\xi_2\xi'_3), & (1+2\omega^2)(\xi_1\xi'_1 + 2\xi_2\xi'_3), & \begin{matrix} 3\xi_1\xi'_1 + 6\xi_2\xi'_3 \\ +6\omega(\xi_1\xi'_3 + \xi_2\xi'_2 + \xi_3\xi'_1) \end{matrix} \end{pmatrix} \tag{3.17}$$

Then, for the vacuum direction

$$\langle\Phi_{11}\rangle = \langle\Phi_{22}\rangle = \langle\Phi_{33}\rangle, \tag{3.18}$$

we obtain a C_3 symmetric mass matrix,

$$\frac{\lambda}{9} \begin{pmatrix} a & c^* & b \\ b & a & c^* \\ c^* & b & a \end{pmatrix}, \tag{3.19}$$

where

$$a = 3\xi_1\xi'_1 + 6\xi_2\xi'_3, \quad b = (1+2\omega^2)(\xi_1\xi'_1 + 2\xi_2\xi'_3), \quad c^* = (1+2\omega)(\xi_1\xi'_1 + 2\xi_2\xi'_3).$$

This is one example how a specific direction of the permutation group is chosen by spontaneous symmetry breaking. In gauge theories, such idea has been extensively studied [17]. For other directions, the relations are not so simple and we do not present them here in detail. Below, we take the viewpoint that the vacuum chooses such directions when we assume a specific form of mass matrix.

4. Mixing matrix of light and heavy neutrinos

To fix the MNS mixing matrix $U_{\rm MNS}$ of (1.7), one needs the neutrino mixing matrix U_ν obtained from (2.9) and charged lepton mixing matrix U_l obtained from (2.1). If we identify the family symmetry ansatz even to Majorana neutrinos, (2.6) = (2.7), the MNS mixing matrix would be identity, leading to no mixing angle between different families. But the 'family' structure defined for Majorana neutrino masses can be in principle different from the family structure defined for Dirac masses. In this spirit, let us assume

$$f^{(lN)} \neq f^{(nn)} \tag{4.1}$$

4.1 n-tuple maximal mixing

The maximal mixing angle in the fit of the atmospheric neutrino data suggests some kind of symmetry. The simple form of mass matrix with the flavor democracy [11] is

$$m \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{4.2}$$

which has one heavy and two massless neutrinos. The above flavor democratic form belongs to a special case of permutation symmetry S_3 which has been extensively studied for neutrino masses [18].

In general, the permutation symmetry of n Majorana neutrinos dictates the following type mass matrix,

$$m_n \propto \begin{pmatrix} 1 & r & r & \cdots & r \\ r & 1 & r & \cdots & r \\ r & r & 1 & \cdots & r \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r & r & r & \cdots & 1 \end{pmatrix}. \tag{4.3}$$

With a real r, we obtain for example the eigenvalues of $(1 \pm r)$ for $n = 2$, two $(1 - r)$ and one $(1 + 2r)$ for $n = 3$, and two $(1 - r)$ and $1 + r(1 \pm \sqrt{2}i)$ for $n = 4$. Since we will be interested in three families, we do not consider the complication arising from $n \geq 4$. For $n = 3$, there exists a hierarchy of Δm_{ij}^2, which is useful in explaining both atmospheric and solar neutrino data. For charged leptons, in general the mass matrix is complex and does not take the form (4.3).

Diagonalizing (4.3), we obtain a *bi-maximal* unitary transformation for the case of $n = 2$ [20],

$$U_{\nu,2\times 2} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.4}$$

m_2 is diagonalized as

$$U^\dagger_{\nu,2\times 2} m_2 U_{\nu,2\times 2} \propto \begin{pmatrix} 1 - r & 0 \\ 0 & 1 + r \end{pmatrix}. \tag{4.5}$$

For $n = 3$, we obtain a *tri-maximal* (third column) unitary transformation,

$$U_{\nu,3\times 3} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{4.6}$$

which diagonalizes m_3,

$$U^\dagger_{\nu,3\times 3} m_3 U_{\nu,3\times 3} \propto \begin{pmatrix} 1-r & 0 & 0 \\ 0 & 1-r & 0 \\ 0 & 0 & 1+2r \end{pmatrix}. \tag{4.7}$$

From (4.4) and (4.6), we try to make a tri-bi-maximal matrix, with one more row and column to be added to (4.4) at our disposal. But with the form (4.6), one cannot obtain a tri-bi-maximal mixing. Using the form (4.6) directly for charged leptons is not correct anyway since the mass matrix m_l itself for charged leptons is not Hermitian. We can consider a Hermitian matrix $m_l^\dagger m_l$ for the diagonalization of which one uses the unitary transformation of left-handed charged leptons U_l.

4.2 Light neutrino mass matrix from heavy neutrino mass matrix

Following the scheme of the previous section, we investigate the mass matrix $m^{(nn)}$ which is proportional to m_ν. The diagonalizing matrix of $m^{(nn)}$ will appear in the neutrino mixing matrix. The flavor democratic form for $m^{(nn)}$ (hence also for m_ν via the double seesaw) introduces one heavy and two massless neutrinos. Therefore, the S_3 symmetric form (4.3) introduces one heavy neutrino and two massive degenerate neutrinos. If the S_3 symmetry is slightly broken by ϵ_n in the degenerate subspace in the following way

$$m^{(nn)} = c \begin{pmatrix} 1-r & 0 & \epsilon_n \\ 0 & 1+2r & 0 \\ \epsilon_n & 0 & 1-r \end{pmatrix}, \tag{4.8}$$

the degenerate mass eigenvalues are split into $(1-r\pm\epsilon_n)$ and $1+2r$. Here, the bi-maximal mixing matrix for diagonalizing $m^{(\nu)}$ is [20],

$$U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.9}$$

But the form (4.9) does not depend on the strength of ϵ_n [20]. In fact, the mass matrix form (4.8) indicates that the neutrino triplet transforms as a singlet(n_2) plus a doublet(n_1 and n_3) since the 2×2 subspace of $m^{(nn)}$ has the structure of the form (4.8) in this subspace. However, we will treat $|\epsilon_n| \ll 1$ so that two scales of Δm^2_{IJ} is obtained such that the atmospheric(2-3 subspace) and solar(1-3 subspace) neutrino data are explained.

So far we discussed the detailed structure of $f^{(nn)}$ in terms of the S_3 permutation symmetry. The form (4.9) can arise from a tiny breaking of the S_3 permutation symmetry. Note that with $\epsilon_n = 0$, we recover the S_3 symmetry in the original basis (4.3).

JHEP05(2006)017

4.3 Charged leptons

On the other hand, the mass hierarchy of charged leptons is quite different from neutrino masses, $m_e \ll m_\mu \ll m_\tau$. Therefore, if S_3 is a useful symmetry, the mass matrix of charged leptons must break S_3 form (4.3) further to have three different masses for charged leptons. Since an S_3 symmetric real mass matrix form (4.3) has a degenerate pair which we try to avoid, we must use a subset of S_3 generators, leading to tri-maximal mixing. One obvious try is the cyclic permutation, i.e. $\{ijk\}$ of eq. (3.2) is a cyclic permutation of (123). Thus, we choose only three elements among six S_3 generators, which is a cyclic permutation in one direction, C_3,

$$P_{123} \equiv I, \quad P_{231} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 2 & 3 & 1 \end{pmatrix}, \quad P_{312} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 3 & 1 & 2 \end{pmatrix}. \tag{4.10}$$

Namely, we violate the exchange symmetry among any two indices. Choosing a subset of generators is achieved by the Higgs mechanism, which we will explore in a future communication. Still we have a subset permutation among *three* in C_3, there is a possibility of tri-maximal mixing. Taking the following mass matrix for charged leptons, consistently with the cyclic permutation C_3,

$$m^{(l)} = \begin{pmatrix} a & c^* & b \\ b & a & c^* \\ c^* & b & a \end{pmatrix}, \tag{4.11}$$

we have

$$M_l^2 \equiv (m^{(l)})^\dagger m^{(l)} = \begin{pmatrix} A & B^* & B \\ B & A & B^* \\ B^* & B & A \end{pmatrix}, \tag{4.12}$$

where

$$A = |a|^2 + |b|^2 + |c|^2, \quad B = a^* b + b^* c^* + ac.$$

Indeed, the diagonalizing matrix U_l turns out to be tri-maximal [4],

$$U_l = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}, \tag{4.13}$$

where ω is a square root of unity, $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. U_l diagonalizes M_l^2 to

$$U_l^\dagger M_l^2 U_l = \begin{pmatrix} A + B + B^* & 0 & 0 \\ 0 & A + B\omega + B^*\omega^2 & 0 \\ 0 & 0 & A + B\omega^2 + B^*\omega \end{pmatrix}. \tag{4.14}$$

Thus, we identify A, B and B^* as

$$A = \frac{1}{3}(m_e^2 + m_\mu^2 + m_\tau^2) \tag{4.15}$$

$$B = \frac{1}{3}(m_e^2 + m_\mu^2\omega^2 + m_\tau^2\omega) \tag{4.16}$$

$$B^* = \frac{1}{3}(m_e^2 + m_\mu^2\omega + m_\tau^2\omega^2) \tag{4.17}$$

4.4 Tri-bi-maximal mixing

Now, the MNS mixing matrix can be expressed in terms of U_ν and U_l,

$$U_{\rm MNS} = U_l^\dagger U_\nu \ , \quad \text{or} \quad U_\nu = U_l U_{\rm MNS} \ . \tag{4.18}$$

From the neutrino mixing (4.9) and the charged lepton mixing (4.13), thus we obtain

$$U_{\rm MNS} = U_l^\dagger U_\nu = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\omega^2}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{2}} \end{pmatrix} . \tag{4.19}$$

Defining $i\nu_3$ as a new mass eigenstate, the desired tri-bi-maximal mixing matrix results

$$U_{\rm MNS} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} . \tag{4.20}$$

5. Relation $\theta_{\rm sol} + \theta_c \simeq \frac{\pi}{4}$

5.1 Up type quark masses

The intriguing phenomenological relation $\theta_{\rm sol}^{\rm exp} + \theta_c^{\rm exp} \simeq \frac{\pi}{4}$ can be explained only if one relates the lepton and quark sectors, which is the basic principle of GUTs. In the quark sector, both the up and the down type mass matrices are complex. We observe the similarity in the hierarchies of charged lepton masses and $Q_{\rm em} = -\frac{1}{3}$ quark masses [21]

$$m_e \simeq \frac{1}{200} m_\mu \sim 0, \quad m_\mu \simeq \frac{1}{17} m_\tau \tag{5.1}$$

$$m_d \simeq \frac{1}{20} m_s \sim 0, \quad m_s \simeq \frac{1}{35} m_b. \tag{5.2}$$

Along with charged leptons, we propose that the complex down type quark mass matrix is C_3 symmetric, leading to three hierarchical masses of 0, $\sim \frac{1}{20} - \frac{1}{35}$ and 1. In GUTs, the small discrepancy between charged lepton and $Q_{\rm em} = -\frac{1}{3}$ quark masses is explained in various ways, for example by introducing the Georgi-Jarlskog type term [22].

But for the up type quarks there is a huge hierarchy of 0, $\frac{1}{150}$ and 1, due to the very large top quark mass

$$m_u \simeq \frac{1}{200} m_c \sim 0, \qquad m_c \simeq \frac{1}{150} m_t. \tag{5.3}$$

Here arises a question, "Should we treat the up type quark mass matrix $m^{(u)}$ as interpreting one heavy and two degenerate zero masses or three nondegenerate masses?" Since any perturbation can add a small addition, it is better to treat the up type matrix $m^{(u)}$ as the first case, namely having one heavy and two zero mass eigenvalues. In addition to this phenomenological observation, treating $Q_{\rm em}=-\frac{1}{3}$ quarks and $Q_{\rm em}=\frac{2}{3}$ quarks differently is required to obtain a nontrivial CKM matrix. Then, it is of the same form as $m^{(nn)}$, but not quite because one is complex and the other is real. The matrix $m^{(u)\dagger}m^{(u)}$ is required to have two zero eigenvalues, which must be done with the S_3 symmetry.

Here we emphasize two aspects: one that the quark mass matrix is complex and another that u quark is almost massless from the outset. Thus we introduce a flavor democratic form or an S_2 symmetric form with $r=\pm 1$ in the 2×2 subspace with zero entries at the other row and column. This type of mass matrix is

$$m^{(u)\dagger}m^{(u)}\propto\begin{pmatrix}0&0&0\\0&1&\pm 1\\0&\pm 1&1\end{pmatrix}\tag{5.4}$$

For an explicit demonstration, we choose the minus sign in eq. (5.4) and obtain the third eigenvalue as m_t with the following diagonalizing unitary matrix

$$U^{(u)}=\begin{pmatrix}1&0&0\\0&\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\\0&\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\end{pmatrix}\tag{5.5}$$

where $U^{(u)\dagger}m^{(u)\dagger}m^{(u)}U^{(u)}=(m^{(u)\dagger}m^{(u)})_{\rm diag}$.

5.2 Relating MNS and CKM angles

The MNS mixing matrix and the CKM mixing matrix are given by

$$\begin{aligned}U_{\rm MNS}&=U_l^\dagger U_\nu,\quad \text{or}\ \ U_l=U_\nu U_{\rm MNS}^\dagger,\\ U_{\rm CKM}&=U^{(u)\dagger}U^{(d)},\quad \text{or}\ \ U^{(d)}=U^{(u)}U_{\rm CKM}.\end{aligned}\tag{5.6}$$

In unifying models, $U^{(d)}$ and U_l are usually related,

$$U^{(d)}=U_l:\ \text{quark-lepton complementarity}\tag{5.7}$$

$$U^{(d)}=U_l^\dagger:\ \text{SU(5) GUT}\tag{5.8}$$

If we choose the quark-lepton complementarity relation, the MNS and CKM angles are related by

$$U_{\rm CKM}U_{\rm MNS}\simeq U^{(u)\dagger}U_\nu:\quad \text{quark-lepton complementarity}\tag{5.9}$$

On the other hand, the SU(5) GUT relation gives

$$U^{(u)}U_{\rm CKM}=U_{\rm MNS}U_\nu^\dagger:\quad \text{SU(5) GUT.}\tag{5.10}$$

The SU(5) GUT relation can be studied with a specific form of $U^{(u)}$ and/or U_ν. Here, we illustrate our idea with the quark-lepton complementarity, (5.9). The LHS of (5.9) relates θ_c and $\theta_{\rm sol}$. Note that $\sin\theta_c^{\rm exp}\simeq 0.22$ which leads to $\theta_c^{\rm exp}\simeq 0.071\pi$, and from $\cos\theta_{\rm sol}^{\rm th}=\frac{\sqrt{2}}{\sqrt{3}}$ we have $\theta_{\rm sol}^{\rm th}\simeq 0.196\pi$; thus $\theta_c^{\rm exp}+\theta_{\rm sol}^{\rm th}\simeq 0.267\pi$. Basically, θ_c and $\theta_{\rm sol}$ are related to U_{11} elements of $U_{\rm CKM}$ and $U_{\rm MNS}$. Thus, we are interested in the first row of a real form of $U_{\rm CKM}$ which are parametrized by two angles θ_c and φ_q

$$U_{\rm CKM}^{\rm th}\simeq\begin{pmatrix}\cos\theta_c^{\rm th} & \sin\theta_c^{\rm th}\cos\varphi_q & \sin\theta_c^{\rm th}\sin\varphi_q\\ \times & \times & \times\\ \times & \times & \times\end{pmatrix}. \tag{5.11}$$

In the same vein, we are interested in the first column of a real form of $U_{\rm MNS}$ which are parametrized by two angles $\theta_{\rm sol}$ and φ_l,

$$U_{\rm MNS}^{\rm th}\simeq\begin{pmatrix}\cos\theta_{\rm sol}^{\rm th} & \times_{12} & 0\\ \sin\theta_{\rm sol}^{\rm th}\cos\varphi_l & \times_{22} & \times_{23}\\ \sin\theta_{\rm sol}^{\rm th}\sin\varphi_l & \times_{32} & \times_{33}\end{pmatrix} \tag{5.12}$$

from which we obtain

$$(U_{\rm CKM}^{\rm th}U_{\rm MNS}^{\rm th})_{11}=\cos\theta_c^{\rm th}\cos\theta_{\rm sol}^{\rm th}+\sin\theta_c^{\rm th}\sin\theta_{\rm sol}^{\rm th}(\cos\varphi_q\cos\varphi_l+\sin\varphi_q\sin\varphi_l).$$

From the tri-bi-maximal form (1.5), we identify $\cos\varphi_l=-\frac{1}{\sqrt{2}}$ and $\sin\varphi_l=-\frac{1}{\sqrt{2}}$, giving $\varphi_l=\frac{5}{4}\pi$. So, we obtain $\cos\varphi_q\cos\varphi_l+\sin\varphi_q\sin\varphi_l=\cos(\varphi_l-\varphi_q)=-\frac{1}{\sqrt{2}}(\cos\varphi_q+\sin\varphi_q)$.

The particle data book [21] gives $U_{\rm CKM\ 11}=(0.9739$ to $0.9751)$ and $U_{\rm CKM\ 13}=(0.0029$ to $0.0045)$, which gives $\varphi_q\simeq(0.0049-0.0065)\pi$. Thus, we have

$$\begin{aligned}\left(U_{\rm CKM}^{\rm th}\ U_{\rm MNS}^{\rm th}\right)_{11}&=\cos(\theta_c^{\rm th}+\theta_{\rm sol}^{\rm th})+\sin\theta_c^{\rm th}\sin\theta_{\rm sol}^{\rm th}[1+\cos(\varphi_l-\varphi_q)]\\ &=\cos(\theta_c^{\rm th}+\theta_{\rm sol}^{\rm th})+\sin\theta_c^{\rm th}\sin\theta_{\rm sol}^{\rm th}\left[1-\cos\left(\frac{\pi}{4}-\varphi_q\right)\right]\\ &\to\cos(\theta_c^{\rm th}+\theta_{\rm sol}^{\rm th})+\sin\theta_c^{\rm exp}\sin\theta_{\rm sol}^{\rm exp}\left[1-\cos\left(\frac{\pi}{4}-\varphi_q\right)\right]\\ &=\cos(\theta_c^{\rm th}+\theta_{\rm sol}^{\rm th})+\sin\theta_c^{\rm exp}\sin\theta_{\rm sol}^{\rm exp}\left[1-\begin{pmatrix}\cos(0.25-0.0049)\pi\\ \cos(0.25-0.0065)\pi\end{pmatrix}\right]\\ &\simeq\cos(\theta_c^{\rm th}+\theta_{\rm sol}^{\rm th})+0.034.\end{aligned} \tag{5.13}$$

where in the third row we used the experimental value for $\sin\theta_c^{\rm th}\sin\theta_{\rm sol}^{\rm th}[\cdots]$ since the replacement $\theta_c^{\rm th}\to\theta_c^{\rm exp}$ would introduce a small extra piece due to the smallness of $\sin\theta_c$.

On the other hand, the RHS of (5.9) with (5.5) is

$$U^{(u)\dagger}U_\nu=\begin{pmatrix}\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2}\\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2}\end{pmatrix}. \tag{5.14}$$

Then, from eqs. (5.13) and (5.14), we obtain

$$\cos(\theta_c^{\rm th} + \theta_{\rm sol}^{\rm th}) \simeq \frac{1}{\sqrt{2}} \tag{5.15}$$

where the accuracy of the sum is 5% in view of eq. (5.13), which is a pretty good approximation. Thus, with the quark-lepton complementarity ansatz we obtain the following approximate relation,

$$\theta_c^{\rm th} + \theta_{\rm sol}^{\rm th} \simeq \frac{\pi}{4}. \tag{5.16}$$

Let us note that we obtained (5.16) with the following understanding:

(a) The possible dependence on the heavy neutrino Yukawa couplings is removed by the family symmetry. Here, the double seesaw mechanism is used.

(b) We used the S_3 symmetry categorically differently for $Q_{\rm em} = 0, -1, -\frac{1}{3}$, and $\frac{2}{3}$ fermions. In particular, for the up type quarks, we use the mass matrix of the form (5.4), leading to one heavy and two zero masses.

(c) We obtain the relation $\theta_c^{\rm th} + \theta_{\rm sol}^{\rm th} \simeq \frac{1}{4}\pi$ only approximately.

(d) The above relation is the one given at the GUT scale.

(e) We just tried to obtain the relation (5.16). It may not satisfy other phenomenological constraints, for example the CKM matrix elements are not produced successfully. To close this kind of loose ends, more elaborate ansatze may be needed rather than the oversimplified quark-lepton universality.

Corrections. Running the Yukawa couplings from the GUT scale down to the electroweak scale can change the relation (5.16) significantly. But there exist ideas that this relation is not renormalized very much [23]. The relation is expected to be renormalized by large Yukawa couplings and the QCD coupling. For the Dirac type Yukawa couplings involving heavy leptons N and n, they are independent from the top quark Yukawa coupling and hence can be taken as small values. So only the top quark Yukawa coupling is important. To use the relation (5.9), the RHS is evaluated at the unification scale and the LHS uses the experimental values at the electroweak scale. So we do not worry about the renormalization of the RHS.

The QCD coupling is flavor blind and hence the correction to quark masses is universal, leading to a factor of 3 modification [10] to quark masses down to 5 GeV, except that for top quark. For top quark, the difference from 3 is $\ln(5/175)/\ln(5/10^{16}) \sim 0.1$. But this correction is not what we are interested in since we use $m^{(u)}$ which has two zero eigenvalues. This structure of $m^{(u)}$ is not changed. For $m^{(d)}$, both s and b quarks are renormalized by the same factor 3, and hence we expect that $U_{\rm CKM}$ is not changed very much by α_s. In particular, we use only $U_{\rm CKM\ 11}$ which is close to 1 before and after the α_s correction [24].

For $m^{(d)}$, the most significant change due to the large top quark Yukawa coupling (symbolically represented as Y_t) is expected to arise from the violation of $m_l = m^{(d)}$ where $m^{(d)}$ is corrected by large Y_t. Both m_l and $m^{(d)}$ are expected to take the form (4.11) at a

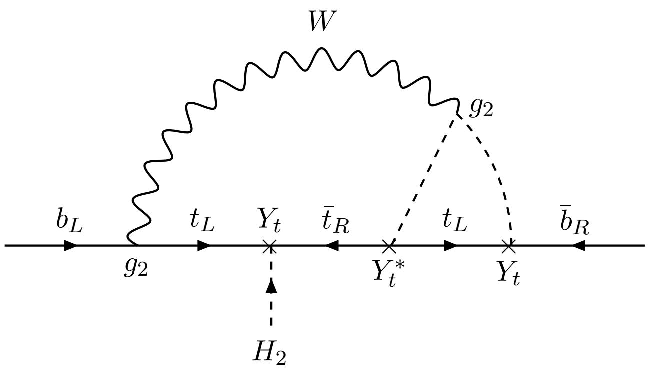

Figure 2: A schematic view of corrections of $m^{(d)}$.

quark-lepton complementary scale. The extra correction to $m^{(d)}$ due to Y_t is expected to arise from the diagrams of the form given in figure 2, whose strength is estimated roughly as

$$\frac{|Y_t|^3 g_2^2}{(4\pi^2)^2} \cdot (\text{kinematic factor}) \sim \frac{|Y_t|^3 \alpha_2}{4\pi^3} \sim 2.5 \times 10^{-4} \tag{5.17}$$

which is smaller than $\frac{1}{20}$ of eq. (5.2). Hence the down type mass category is not drastically changed, and hence the CKM angles are not changed drastically.

Most studies on the correction of MNS angles in GUTs have been performed by studying the running of neutrino masses arising through the dimension-5 operators llH_2H_2 [23]. Here, one usually assumes a large Yukawa coupling in view of the large top mass. Then, for non-hierarchical neutrino masses the MNS angle is known to go a drastic change, and for hierarchical neutrino masses the correction remains negligible. But in our double seesaw, the needed Yukawa couplings $f_{IJ}^{(lN)}$ and $f_{IJ}^{(Nn)}$ (viz. eq. (2.9)) can be taken to be small, and running of the MNS angle can be made negligible by taking $|f_{IJ}^{(lN)}| \ll 1$.

Therefore, we expect that the LHS of (5.9) is not corrected very much by going to the electroweak scale, and hence the sum (5.16) is still valid.

Z_3 **orbifolds.** The S_3 symmetry we discuss is expected to arise from a more fundamental theory. In the framework of quantum field theory, we can dictate relevant couplings corresponding to the presumed family symmetry. But it is our hope to obtain the couplings from a more fundamental theory. One example is string theory. From string theory, a good example allowing the permutation symmetry of 3 objects is the Z_3 orbifold compactification of $E_8 \times E_8'$ heterotic string [25]. The Yukawa couplings resulting from a Z_3 orbifold do not know how to distinguish the difference of three objects, leading to the discrete symmetry. The specific forms for Yukawa couplings are the result of spontaneous symmetry breaking of S_3 symmetry. We note that in Z_3 orbifolds, the $\sin^2\theta_W$ problem hints toward a trinification model [16], in which we will explore a realization of the mass matrices discussed here in a future communication.

6. Conclusion

In this paper, we use the family symmetry S_3 which is dictated to be realized differently for $Q_{\rm em} = 0, -1, -\frac{1}{3}$, and $\frac{2}{3}$ fermions. We introduce two types of heavy neutral leptons n and N with two additional continuous symmetries F_1 and F_2. To discuss neutrino masses just from the symmetry principle, it is suggested to use the double seesaw mechanism so that the Yukawa couplings of N and lepton doublets are removed. The double seesaw diagram of figure 1 removes the Yukawa coupling dependence if VEVs of singlets have a hierarchy $\langle S_2\rangle \gg \langle S_1\rangle \gg \langle H_2\rangle$. Then one obtains a direct proportionality between the neutrino mass matrix and the n type Majorana mass matrix, viz. eq. (2.9). Now it becomes possible to discuss just the mass matrices, and we note that neutrinos, charged leptons, and $Q_{\rm em} = -\frac{1}{3}, \frac{2}{3}$ quarks have distinct patterns of mass hierarchy. In the S_3 symmetric scheme, the mass matrix forms of n, charged leptons and $Q_{\rm em} = -\frac{1}{3}, \frac{2}{3}$ quarks are dictated to be realized to conform with the observed mass patterns. These different patterns are the source of nontrivial MNS and CKM angles. A realization of these different patterns is expected to result from spontaneous symmetry breaking of family symmetry. In quark and lepton unification models, some of these angles can be related. In this paper, we studied the quark-lepton complementarity to relate the charged lepton type mixing matrix and the down type quark mixing matrix. The $SU(5)$ GUT type relation is also possible, for which however the resulting relation is not so simple. Finally, we also suggested a way to understand the approximate relation $\theta_c^{\rm th} + \theta_{\rm sol}^{\rm th} \simeq \frac{\pi}{4}$.

Acknowledgments

We thank E.J. Chun, S.K. Kang and S. Raby for useful discussions. This work is supported in part by the KRF Sundo Grant No. R02-2004-000-10149-0, the KRF ABRL Grant No. R14-2003-012-01001-0, and the BK21 program of Ministry of Education.

References

[1] See, for example, M. Fukugita and T. Yanagida, *Physics of neutrinos and application to astrophysics*, Springer-Verlarg, Berlin, 2003, p. 327.

[2] S.P. Mikheev and A.Y. Smirnov, *Resonance enhancement of oscillations in matter and solar neutrino spectroscopy*, *Sov. J. Nucl. Phys.* **42** (1985) 913;
L. Wolfenstein, *Neutrino oscillations in matter*, *Phys. Rev.* **D 17** (1978) 2369.

[3] L. Wolfenstein, *Oscillations among three neutrino types and CP-violation*, *Phys. Rev.* **D 18** (1978) 958.

[4] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett.* **B 530** (2002) 167 [hep-ph/0202074].

[5] E. Ma, *Tetrahedral family symmetry and the neutrino mixing matrix*, *Mod. Phys. Lett.* **A 20** (2005) 2601 [hep-ph/0508099]; *Pattern of the approximate mass degeneracy of majorana neutrinos*, *Mod. Phys. Lett.* **A 17** (2002) 289 [hep-ph/0201225]; *Quark mass matrices in the A_4 model*, *Mod. Phys. Lett.* **A 17** (2002) 627 [hep-ph/0203238]; *A_4 origin of the neutrino mass matrix*, *Phys. Rev.* **D 70** (2004) 031901 [hep-ph/0404199];

A. Zee, *Obtaining the neutrino mixing matrix with the tetrahedral group*, *Phys. Lett.* **B 630** (2005) 58 [hep-ph/0508278];
S. Luo and Z.-z. Xing, *Generalized tri-bimaximal neutrino mixing and its sensitivity to radiative corrections*, *Phys. Lett.* **B 632** (2006) 341 [hep-ph/0509065].

[6] C. Giunti and M. Tanimoto, *CP-violation in bilarge lepton mixing*, *Phys. Rev.* **D 66** (2002) 113006 [hep-ph/0209169];
P.H. Frampton, S.T. Petcov and W. Rodejohann, *On deviations from bimaximal neutrino mixing*, *Nucl. Phys.* **B 687** (2004) 31 [hep-ph/0401206];
C.H. Albright, *Bounds on the neutrino mixing angles for an SO(10) model with lopsided mass matrices*, *Phys. Rev.* **D 72** (2005) 013001 [hep-ph/0502161];
A. Datta, L. Everett and P. Ramond, *Cabibbo haze in lepton mixing*, *Phys. Lett.* **B 620** (2005) 42 [hep-ph/0503222];
S. Antusch and S.F. King, *Charged lepton corrections to neutrino mixing angles and CP phases revisited*, *Phys. Lett.* **B 631** (2005) 42 [hep-ph/0508044];
F. Plentinger and W. Rodejohann, *Deviations from tribimaximal neutrino mixing*, *Phys. Lett.* **B 625** (2005) 264 [hep-ph/0507143].

[7] A.Y. Smirnov, *Neutrinos: '... annus mirabilis'*, hep-ph/0402264;
M. Raidal, *Relation between the neutrino and quark mixing angles and grand unification*, *Phys. Rev. Lett.* **93** (2004) 161801 [hep-ph/0404046];
H. Minakata and A.Y. Smirnov, *Neutrino mixing and quark lepton complementarity*, *Phys. Rev.* **D 70** (2004) 073009 [hep-ph/0405088];
P.H. Frampton and R.N. Mohapatra, *Possible gauge theoretic origin for quark-lepton complementarity*, *JHEP* **01** (2005) 025 [hep-ph/0407139];
S.K. Kang, C.S. Kim and J. Lee, *Importance of threshold corrections in quark-lepton complementarity*, *Phys. Lett.* **B 619** (129) 2005 [hep-ph/0501029];
J. Ferrandis and S. Pakvasa, *Quark-lepton complementarity relation and neutrino mass hierarchy*, *Phys. Rev.* **D 71** (2005) 033004 [hep-ph/0412038];
S. Antusch, S.F. King and R.N. Mohapatra, *Quark lepton complementarity in unified theories*, *Phys. Lett.* **B 618** (2005) 150 [hep-ph/0504007];
S.F. King, *Predicting neutrino parameters from SO(3) family symmetry and quark-lepton unification*, *JHEP* **08** (2005) 105 [hep-ph/0506297];
M. Lindner, M.A. Schmidt and A.Y. Smirnov, *Screening of Dirac flavor structure in the seesaw and neutrino mixing*, *JHEP* **07** (2005) 048 [hep-ph/0505067];
T. Ohlsson, *Bimaximal fermion mixing from the quark and leptonic mixing matrices*, *Phys. Lett.* **B 622** (2005) 159 [hep-ph/0506094];
H. Minakata, *Quark-lepton complementarity: a review*, hep-ph/0505262.

[8] M. Lindener et. al., in ref. [7].

[9] Z.-z. Xing, *Nontrivial correlation between the CKM and MNS matrices*, *Phys. Lett.* **B 618** (2005) 141 [hep-ph/0503200];
C. Jarlskog, *Ambiguities pertaining to quark-lepton complementarity*, *Phys. Lett.* **B 625** (2005) 63 [hep-ph/0507212].

[10] A.J. Buras, J.R. Ellis, M.K. Gaillard and D.V. Nanopoulos, *Aspects of the grand unification of strong, weak and electromagnetic interactions*, *Nucl. Phys.* **B 135** (1978) 66.

[11] H. Fritzsch and Z.-Z. Xing, *Lepton mass hierarchy and neutrino oscillations*, *Phys. Lett.* **B 372** (1996) 265 [hep-ph/9509389]; *Large leptonic flavor mixing and the mass spectrum of*

leptons, *Phys. Lett.* **B 440** (1998) 313 [hep-ph/9808272]; *Maximal neutrino mixing and maximal CP-violation*, *Phys. Rev.* **D 61** (2000) 073016 [hep-ph/9909304];
E.K. Akhmedov, G.C. Branco, F.R. Joaquim and J.I. Silva-Marcos, *Neutrino masses and mixing with seesaw mechanism and universal breaking of extended democracy*, *Phys. Lett.* **B 498** (2001) 237 [hep-ph/0008010];
R. Dermisek, *Neutrino masses and mixing, quark lepton symmetry and strong right-handed neutrino hierarchy*, *Phys. Rev.* **D 70** (2004) 073016 [hep-ph/0406017];
H. Fritzsch and Z.Z. Xing, *Democratic neutrino mixing reexamined*, *Phys. Lett.* **B 598** (2004) 237 [hep-ph/0406206].

[12] R.N. Mohapatra, *Mechanism for understanding small neutrino mass in superstring theories*, *Phys. Rev. Lett.* **56** (1986) 561;
R.N. Mohapatra and J.W.F. Valle, *Neutrino mass and baryon-number nonconservation in superstring models*, *Phys. Rev.* **D 34** (1986) 1642;
For recent discussion, see for example, T. Blazek, S. Raby and K. Tobe, *Neutrino oscillations in a predictive SUSY GUT*, *Phys. Rev.* **D 60** (1999) 113001 [hep-ph/9903340];
T. Blazek, S. Raby and K. Tobe, *Neutrino oscillations in an SO(10) SUSY GUT with* $\mathrm{U}(2)\times\mathrm{U}(1)^n$ *family symmetry*, *Phys. Rev.* **D 62** (2000) 055001 [hep-ph/9912482];
R. Dermisek and S. Raby, *Fermion masses and neutrino oscillations in SO(10) SUSY GUT with* $D(3)\times\mathrm{U}(1)$ *family symmetry*, *Phys. Rev.* **D 62** (2000) 015007 [hep-ph/9911275];
S. Raby, *A natural framework for bi-large neutrino mixing*, *Phys. Lett.* **B 561** (2003) 119 [hep-ph/0302027].

[13] R. Barbieri, L.J. Hall, S. Raby and A. Romanino, *Unified theories with U(2) flavor symmetry*, *Nucl. Phys.* **B 493** (1997) 3 [hep-ph/9610449].

[14] F. Gursey, P. Ramond and P. Sikivie, *A universal gauge theory model based on* E_6, *Phys. Lett.* **B 60** (1976) 177.

[15] S. L. Glashow, *Trinification of all elementary particle forces*, in Proc. IV Workshop on Grand Unification, K. Kang et. al. eds., World Scientific, Singapore, 1985, p. 88;
J.E. Kim, *SU(3) trits of orbifolded* $E_8\times E'_8$ *heterotic string and supersymmetric standard model*, *JHEP* **08** (2003) 010 [hep-ph/0308064].

[16] J. E. Kim, Z_3 *orbifold construction of* $SU(3)^3$ *GUT with* $\sin^2\theta_W=\frac{3}{8}$, *Phys. Lett.* **B 564** (2003) 35 [hep-th/0301177];
J.E. Kim, *Trinification with* $\sin^2\theta_W=\frac{3}{8}$ *and seesaw neutrino mass*, *Phys. Lett.* **B 591** (2004) 119 [hep-ph/0403196];
J.E. Kim, *Dynamical* μ *and MSSM*, *JHEP* **06** (2005) 076 [hep-th/0503031];
K.-S. Choi and J.E. Kim, *Three family* Z_3 *orbifold trinification, MSSM and doublet-triplet splitting problem*, *Phys. Lett.* **B 567** (2003) 87 [hep-ph/0305002];
K.S. Choi, K.Y. Choi, K.W. Hwang and J.E. Kim, *Higgsino mass matrix ansatz for MSSM*, *Phys. Lett.* **B 579** (2004) 165 [hep-ph/0308160].

[17] L.F. Li, *Group theory of the spontaneously broken gauge symmetries*, *Phys. Rev.* **D 9** (1974) 1723.

[18] S. Pakvasa and H. Sugawara, *Discrete symmetry and Cabibbo angle*, *Phys. Lett.* **B 73** (1978) 61;
G. Segré and H.A. Weldon, *Natural suppression of strong p and t violations and calculable mixing angles in* $\mathrm{SU}(2)\times\mathrm{U}(1)$, *Phys. Rev. Lett.* **42** (1979) 1191;
P.F. Harrison, D.H. Perkins and W.G. Scott, *Threefold maximal lepton mixing and the solar and atmospheric neutrino deficits*, *Phys. Lett.* **B 349** (1995) 137;

K. Kang, J.E. Kim and P. Ko, *A simple modification of the maximal mixing scenario for three light neutrinos*, *Z. Physik* **C 72** (1996) 671 [hep-ph/9503436].

[19] For an introduction, see for example, E. Ma, *Non-abelian discrete family symmetries of leptons and quarks*, hep-ph/0409075;
N.G. Deshpande, M. Gupta and P.B. Pal, *Flavor changing processes and CP-violation in $S_3 \times Z_3$ model*, *Phys. Rev.* **D 45** (1992) 953.

[20] K. Kang, S.K. Kang, J.E. Kim and P.W. Ko, *Almost maximally broken permutation symmetry for neutrino mass matrix*, *Mod. Phys. Lett.* **A 12** (1997) 1175 [hep-ph/9611369].

[21] Particle Data Group collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.

[22] H. Georgi and C. Jarlskog, *A new lepton-quark mass relation in a unified theory*, *Phys. Lett.* **B 86** (1979) 297.

[23] H. Arason et. al., *Renormalization-group study of the standard model and its extensions: the standard model*, *Phys. Rev.* **D 46** (1996) 3945;
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *General RG equations for physical neutrino parameters and their phenomenological implications*, *Nucl. Phys.* **B 573** (2000) 652 [hep-ph/9910420];
P.H. Chankowski and S. Pokorski, *Quantum corrections to neutrino masses and mixing angles*, *Int. J. Mod. Phys.* **A 17** (2002) 575 [hep-ph/0110249];
S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Running neutrino masses, mixings and CP phases: analytical results and phenomenological consequences*, *Nucl. Phys.* **B 674** (2003) 401 [hep-ph/0305273];
S. Antusch, J. Kersten, M. Lindner, M. Ratz and M.A. Schmidt, *Running neutrino mass parameters in see-saw scenarios*, *JHEP* **03** (2005) 024 [hep-ph/0501272];
J.R. Ellis, A. Hektor, M. Kadastik, K. Kannike and M. Raidal, *Running of low-energy neutrino masses, mixing angles and CP-violation*, *Phys. Lett.* **B 631** (2005) 32 [hep-ph/0506122];
S. Luo and Z.Z. Xing, *Generalized tri-bimaximal neutrino mixing and its sensitivity to radiative corrections*, *Phys. Lett.* **B 632** (2006) 341 [hep-ph/0509065].

[24] H. Arason et. al., ref. [23].

[25] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds*, *Nucl. Phys.* **B 261** (1985) 678; *Strings on orbifolds, 2*, *Nucl. Phys.* **B 274** (1986) 285;
L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Orbifolds and Wilson lines*, *Phys. Lett.* **B 187** (1987) 25;
L.E. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, *Orbifold compactifications with three families of* $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^n$, *Phys. Lett.* **B 191** (1987) 282;
J.A. Casas and C. Muñoz, *Three generation* $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_y$ *models from orbifolds*, *Phys. Lett.* **B 214** (1988) 63.

JHEP05(2006)017